1965 Nobel Prize in Physics: Tomonaga, Schwinger, Feynman

Chirag Gokani
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University of Texas at Dallas

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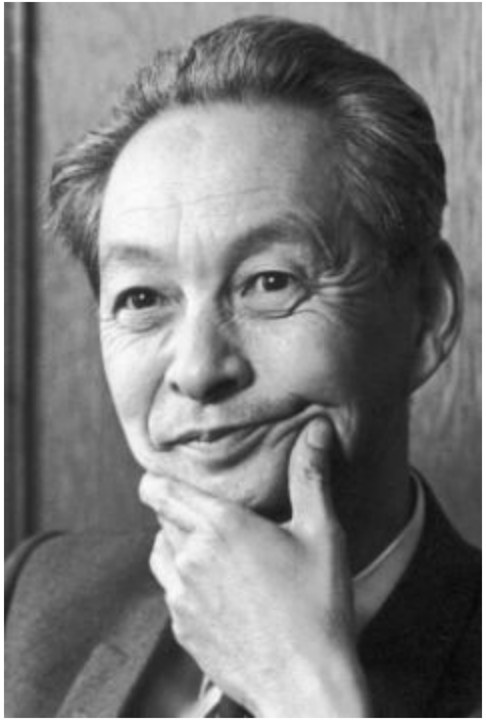
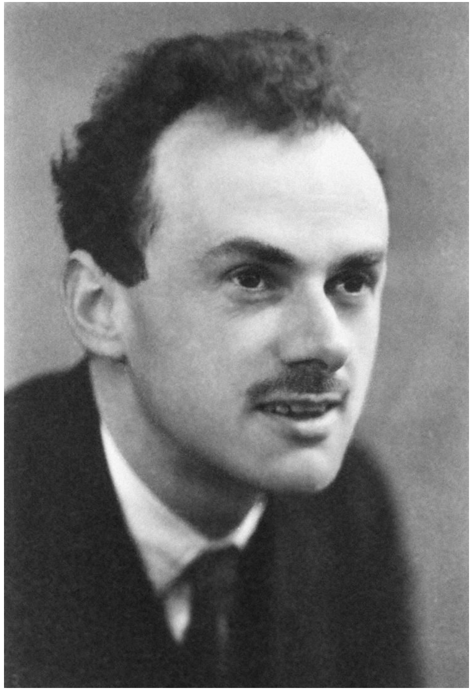
...for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles.



$$(\Box + \mu^2)\psi = 0$$



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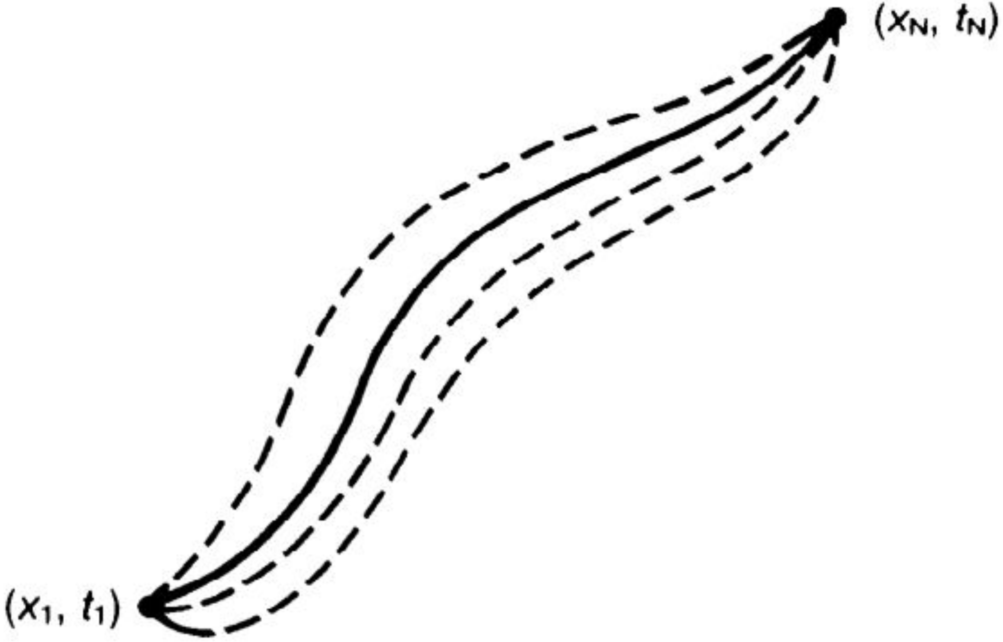
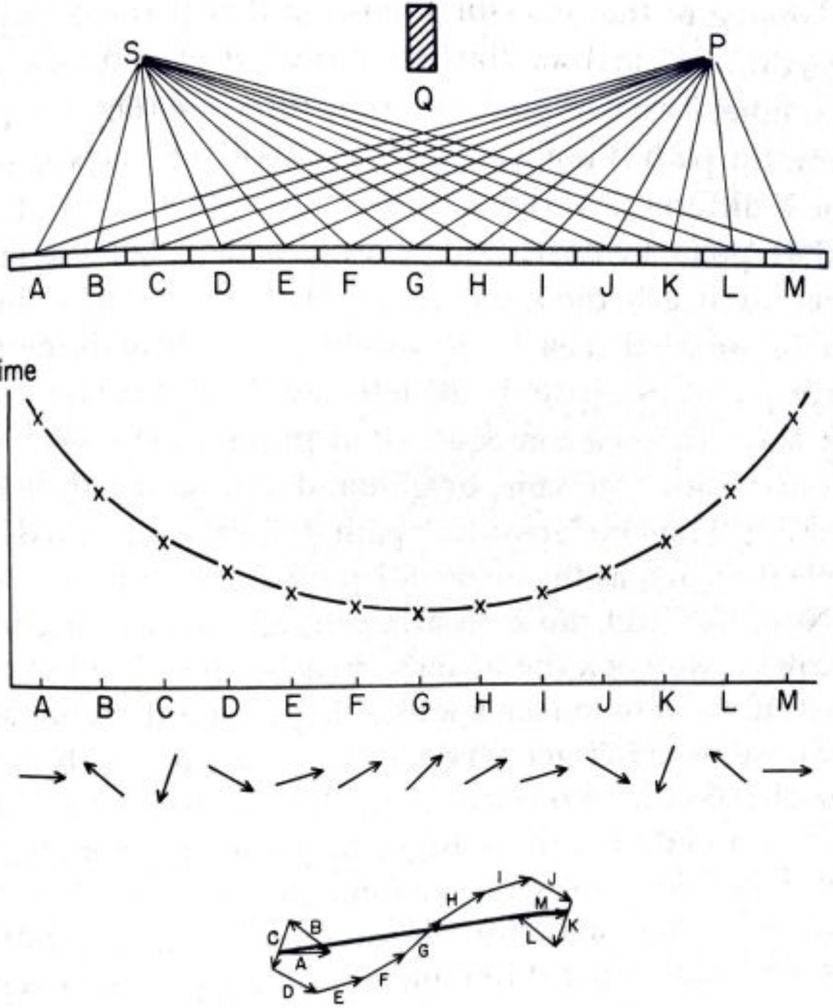
∞

Schrödinger... $$i\hbar\frac{\partial}{\partial t}\Psi(\vec{x}',t) = -\left(\frac{\hbar^2}{2m}\right)\vec{\nabla}'^2\Psi(\vec{x}',t) + V(\vec{x}')\Psi(\vec{x}',t)$$

Heisenberg... $$\frac{\mathrm{d}A(t)}{\mathrm{d}t} = \frac{i}{\hbar}[H, A(t)] + \frac{\partial A}{\partial t}$$

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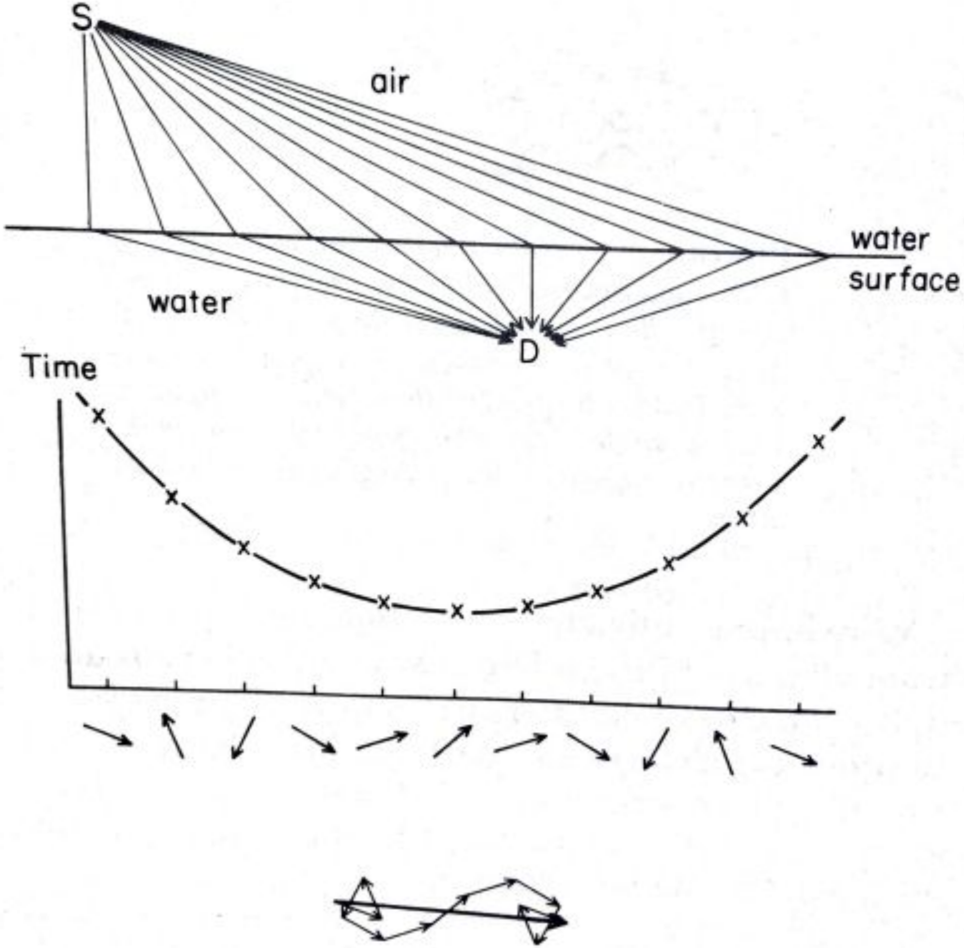
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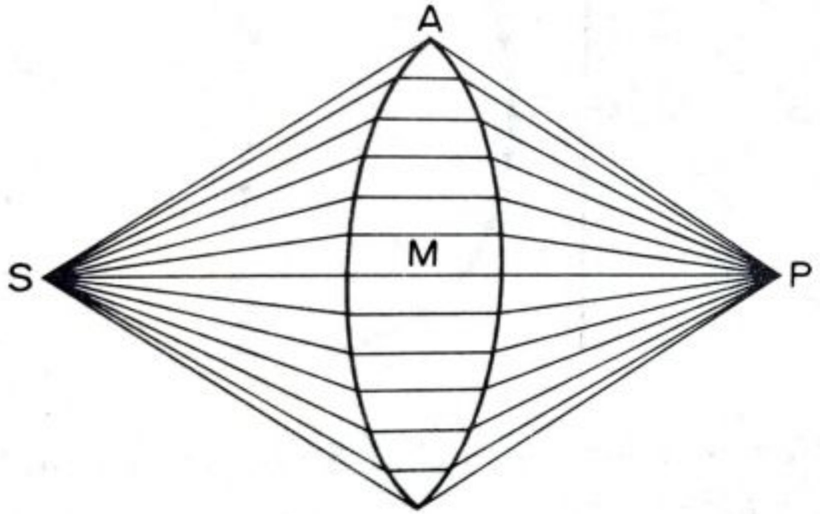
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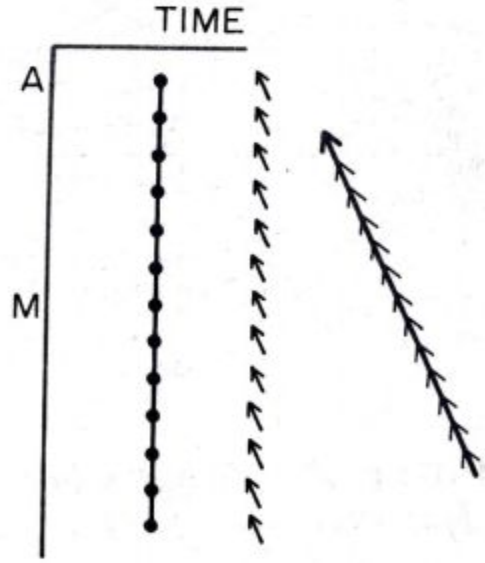
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$$\exp\left(i\int_{t_1}^{t_2}\frac{\mathrm{d}t L_{\text{classical}}(x,\dot{x})}{\hbar}\right) \text{ corresponds to } \langle x_2,t_2|x_1,t_1\rangle$$

Calling $S(n,n-1)\equiv\int_{t_{n-1}}^{t_n}\mathrm{d}t L_{\text{classical}}(x,\dot{x})$, we can apply successive Dirac-like expressions that somehow contributes to $\langle x_N,t_N|x_1,t_1\rangle$:

$$\prod_{n=2}^{N}\exp\left(\frac{iS(n,n-1)}{\hbar}\right)=\cdots=\exp\left(\frac{iS(N,1)}{\hbar}\right)$$

Feynman noted that the classical path is recovered in the $\hbar\to 0$ limit, leading him to conjecture

$$\langle x_N,t_N|x_1,t_1\rangle\sim\sum_{\text{all paths }(A,B,\dots)}\exp\left(\frac{iS(N,1)}{\hbar}\right)$$

Inserting a weighting factor $\frac{1}{w(\Delta t)}$ and considering the case that the time interval t_n-t_{n-1} is infinitesimally small,

$$\langle x_n,t_n|x_{n-1},t_{n-1}\rangle=\frac{1}{w(\Delta t)}\exp\left(\frac{iS(n,n-1)}{\hbar}\right)\tag{3}$$

$$\langle x_N, t_N | x_1, t_1 \rangle = \int_{x_1}^{x_N} \mathcal{D}(x(t)) \exp\left(i \int_{t_1}^{t_N} \mathrm{d}t \frac{L_{\text{classical}}(x, \dot{x})}{\hbar} \right)$$

$$\int_{x_1}^{x_N} \mathcal{D}(x(t)) = \lim_{N \to \infty} \left(\frac{m}{2\pi i \hbar \Delta t} \right)^{\frac{N-1}{2}} \int \mathrm{d}x_{N-1} \int \mathrm{d}x_{N-2} \ldots \int \mathrm{d}x_2$$

If a photon exists at point A, given by the spacetime coordinates (x_1, y_1, z_1) at time t_1, its probability of going to point B, given by the spacetime coordinates (x_2, y_2, z_2) at time t_2, is inversely proportional to the spacetime interval[7]

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - (t_2 - t_1)^2 \tag{8}$$

This probability is often denoted $P(A \text{ to } B)$ ("P" for "photon").

If an electron exists at point A, again given by the spacetime coordinates (x_1, y_1, z_1) at time t_1, its probability of going to point B, again given by the spacetime coordinates (x_2, y_2, z_2) at time t_2, is also related to the inverse of equation (8), *as well as* a number that we will call n. This is an empirically found number that helps us match the theory to experiment. This probability is often denoted $E(A \text{ to } B)$ ("E" for "electrons").

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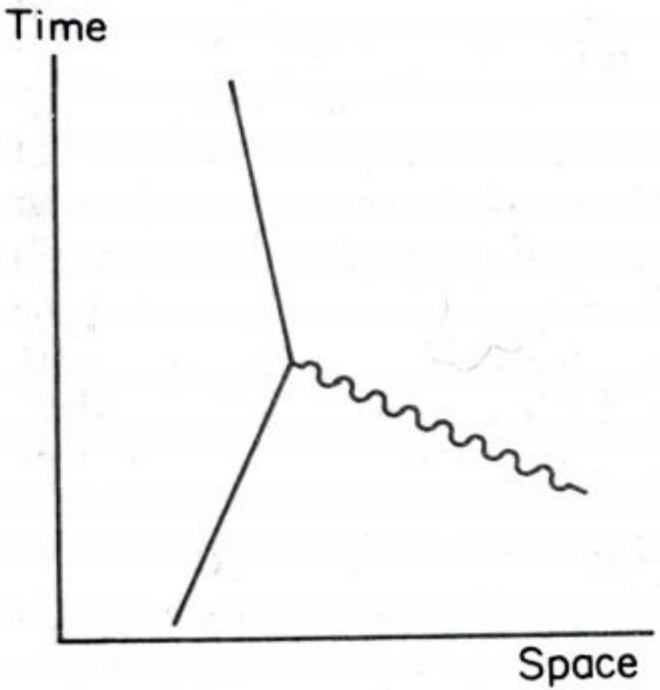

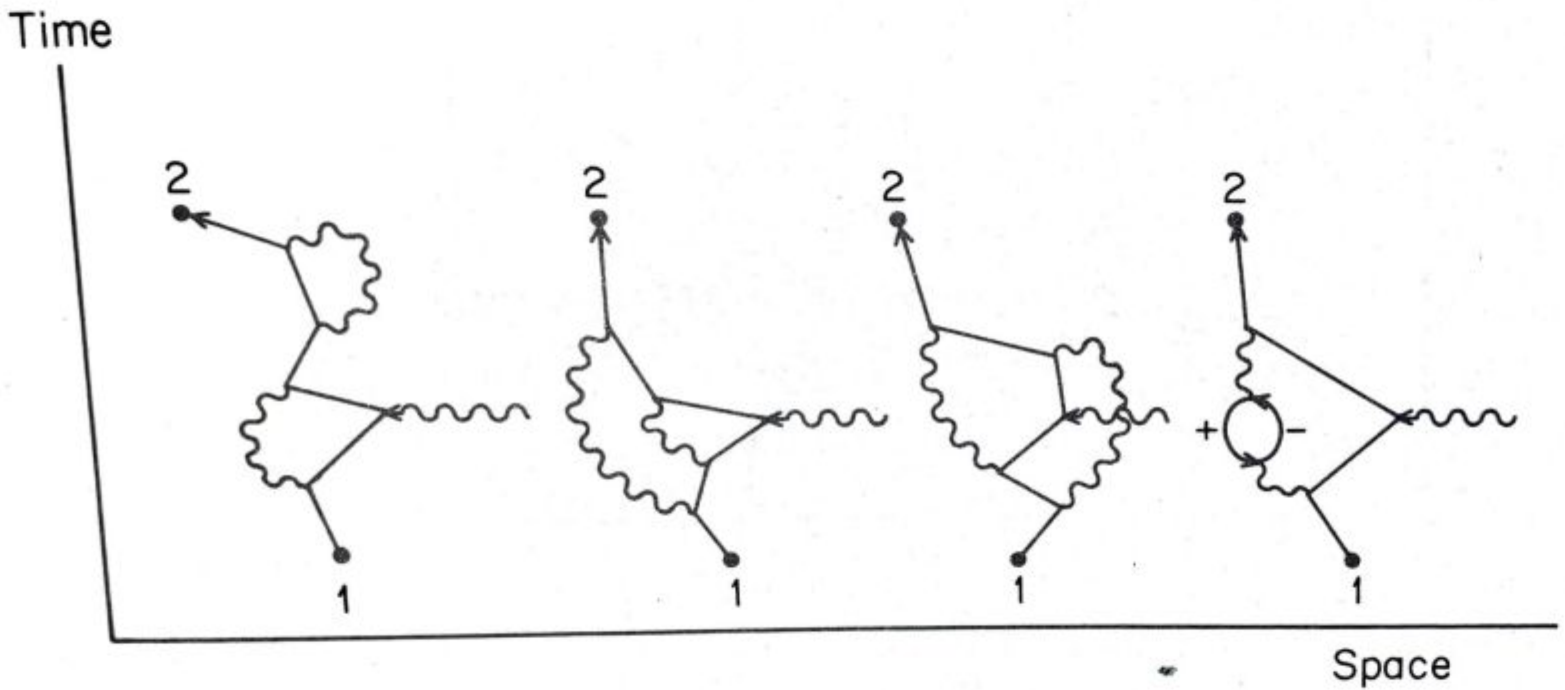

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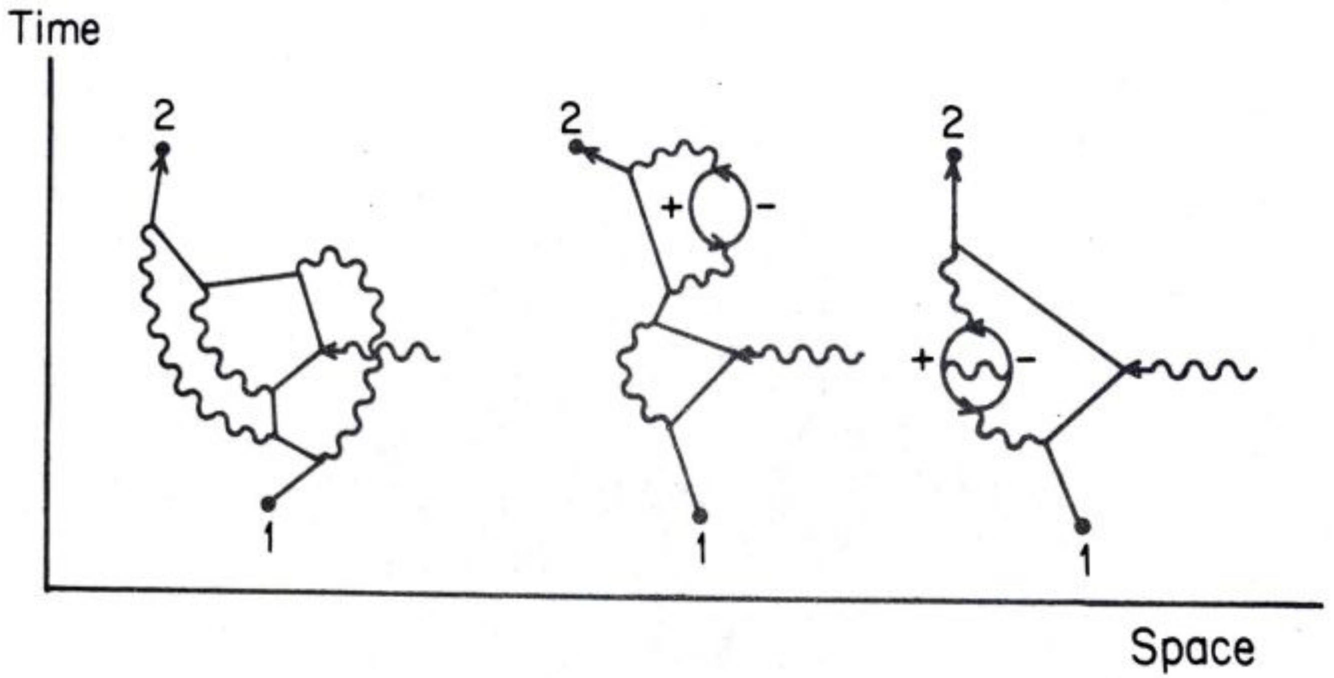
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$$1.00115965221$$

Feynman notes: "If you were to measure the distance from Los Angeles to New York to this accuracy, it would be exact to the thickness of a human hair."[11] Since Feynman's day, this error has only decreased, making QED the most accurate physical theory.

References

David Griffiths. *Introduction to Quantum Mechanics*, 2nd ed. Pearson. 2015.

J.J. Sakurai. *Modern Quantum Mechanics*, 2nd ed. Cambridge University Press. 2017.

Richard P. Feynman. *QED: The Strange Theory of Light and Matter*. Princeton Univerity Press. 1985.