1965 Nobel Prize in Physics: Tomonaga, Schwinger, Feynman

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. . . for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles.

1 Abstract

In this paper, I will outline the subject of the 1965 Nobel Prize in Physics—renormalization—from both a historical and mathematical perspective. To make this rather ad-hoc, mathematical subject tractable, I will devote a significant portion of this paper describing the overall theory of quantum electrodynamics and will then explain why renormalization was necessary. Renormalization succeeded in making quantum electrodynamics a physically testable theory, so a brief summary of later experimental verification will be discussed in section (8).

Since we have already discussed quantum electrodynamics (QED) in the *Concepts of Modern Physics* lecture[1] from the mathematical perspective of symmetry, this paper and the corresponding presentation will focus on the physical inspiration and mechanics of the theory. I will assume the knowledge level of my fellow fourth-year physics undergraduates and will not attempt to re-derive results of classical electrodynamics, non-relativistic quantum mechanics, and Lagrange's formulation.

2 Relevance

This topic is rewarding in that it combines the major themes of *Concepts of Modern Physics* into a single, elegant theory. We began our course by discussing three important numbers—$\hbar$, c, and G—and how they determine the relevance of various regimes of physics over given scales. The course was structured to explore the theory and applications within these regimes; it is fitting for me to end the course by showing the compatibility of the regimes.

We began by reviewing the undergraduate quantum mechanics curriculum, which is applicable for $m \sim \frac{p^2}{\hbar\omega}$ and $v \ll c$. We developed an understanding of how ensembles of bosons and

[1]This was the topic presented by Dr. Shi on November 3rd.

fermions behave in this regime, explaining a multitude of phenomena (Bose-Einstein condensation, lasers, semiconductors, etc.) central to today's physics research and technological applications.

We then shifted our focus to classical special relatively, which is applicable in the limit that $\hbar \to 0$ and $c > v \gg 0$. Here we showed that four-vectors are invariant on the Lorentz transformation, and we discussed classic problems in special relativity (twin paradox, barn-and-ladder paradox, etc.). We also discussed gauge invariance, which, using the Lorentz gauge, allowed us to write Maxwell's equations as a single, elegant equation in relativistic form:

$$\Box A^\mu = -\mu_0 J^\mu$$

In a region without charge and current ($J^\mu = 0$), Maxwell's equations read

$$\Box A^\mu = 0 \tag{1}$$

making it clear that their solution is transverse, spherical waves propagating at speed $\frac{1}{\sqrt{\mu_0 \epsilon_0}} = c$.

Towards the end of the semester, we put together quantum mechanics and special relativity, which applies in the limit $m \sim \frac{p^2}{\hbar\omega}$ and $c > v \gg 0$. For simplicity, we considered a free ($V \to 0$), spinless ($s = 0$) particle. The result was the Klein-Gordon equation:

$$(\Box + \mu^2)\psi = 0 \tag{2}$$

For massless particles ($m = 0$), the reduced mass $\mu = 0$ too, so the Klein-Gordon equation adopts the form of equation (1), the wave equation. So this limit of relativistic quantum mechanics predicts the same spherical wave solution as classical electrodynamics! P.A.M. Dirac sought a more general equation than (2), leading to a more explicit relation of quantum mechanics and special relativity to electrodynamics.

3 Context

In 1929, Dirac arrived at the more generalized version of equation (2), accounting for particles with spin $\frac{1}{2}$, but this equation led to bizarre consequences. Dirac knew that the magnetic moment of an electron should be close to 1 in convenient units (a 1948 experiment corrected this value to 1.00118, not exactly 1 since electrons interact with light). But his theory gave an obviously incorrect value for the magnetic moment: ∞.

While Dirac's theory was aesthetically beautiful, it would be worthless until it matched up with experiment. In 1948, Julian Schwinger, Sin-Itiro Tomaonaga, and Richard Feynman independently sorted out this problem, using what Feynman called a "shell game," for which the three would win the 1965 Nobel Prize.

In sections (4) and (5), I will describe how Dirac inspired Feynman to think of previously well-explained problems and eventually reformulate quantum mechanics using the least action principle. In section (6), I will apply Feynman's formalism to the three actions of QED and describe the need for renormalization.

4 Probability Amplitudes

In the late 1920s, quantum mechanics was predominantly studied in either the

$$\text{Schrödinger...} \qquad i\hbar\tfrac{\partial}{\partial t}\Psi(\vec{x}',t) = -\left(\frac{\hbar^2}{2m}\right)\vec{\nabla}'^2\Psi(\vec{x}',t) + V(\vec{x}')\Psi(\vec{x}',t)$$

$$\text{Heisenberg...} \qquad \tfrac{\mathrm{d}A(t)}{\mathrm{d}t} = \frac{i}{\hbar}[H, A(t)] + \tfrac{\partial A}{\partial t}$$

pictures. Dirac (who had already drawn many parallels between classical and quantum mechanics) began exploring a quantum analog to the classical notion of the "path."

Whether using wave functions in the Schrödinger picture or time-independent operators in the Heisenberg picture, we are accustomed to working with complex numbers in the Hilbert space.[2] To make the notion of the "path" more evident, Feynman suggested representing states using arrows and stopwatches. The length of the arrow represents the magnitude of (working in the Schrödinger picture) the wave function $|\Psi|$, while the stopwatch captures the time dependence $e^{i\omega t}$ (making one rotation in time $\frac{1}{\omega}$). One possible path, then, corresponds to an arrow (of length that, when squared, is proportional to the probability of the particle following that path) pointing in the same direction that the stopwatch points at the end of the trip.

The overall "path" is then just the head-to-tail sum of all the arrows. To illustrate this mechanism, let us see how this model successfully predicts the phenomena of reflection and refraction:

4.1 Reflection

Rather than dictating that photons "travel in straight lines," "travel such that the incident and reflected angles are equal," etc., QED says that the photons *actually* take an infinite number of paths on their way from S to P, as depicted below.[3] The opaque object Q simply blocks photons from transmitting directly from S to P:

[2] David Griffiths. *Introduction to Quantum Mechanics*, 2nd ed. Pearson. 2015. 25-26.
[3] Richard P. Feynman. *QED: The Strange Theory of Light and Matter*. Princeton Univerity Press. 1985.

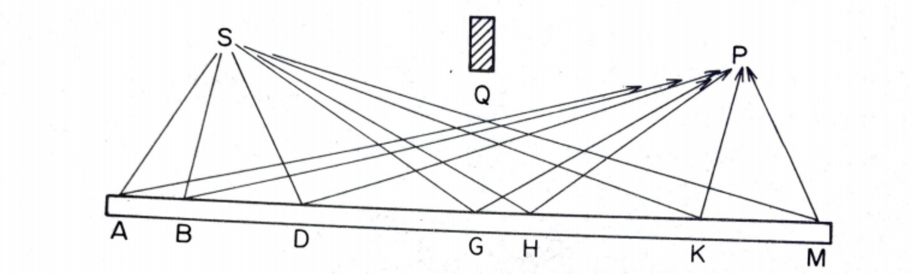
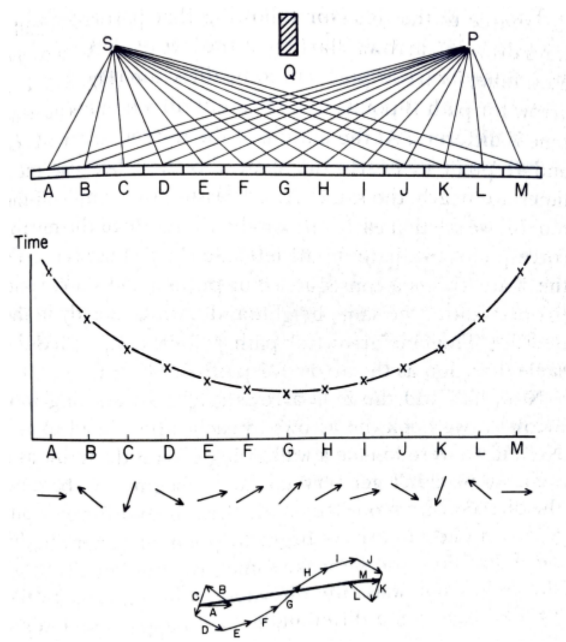


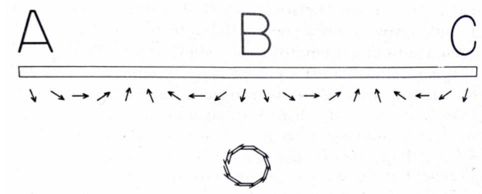
FIGURE 20. *The quantum view of the world says that light has an equal amplitude to reflect from every part of the mirror, from A to M.*

Each of the paths (labeled $A, B, \ldots, M$) is equally likely (so we use arrows of the same size to represent each path), but each path takes a different amount of time to complete the trip (so the final stopwatch readings are different). The arrows point in the direction of the stopwatch readings, as shown below:



In the overall sum above, we see that the biggest contribution to the final arrow's path is due to arrows E through I. They point in almost the same direction because their travel time is almost the same. The accumulation of photons along this path leads us to believe that light only travels in straight lines, and that the angle of incidence equals the angle of reflection.

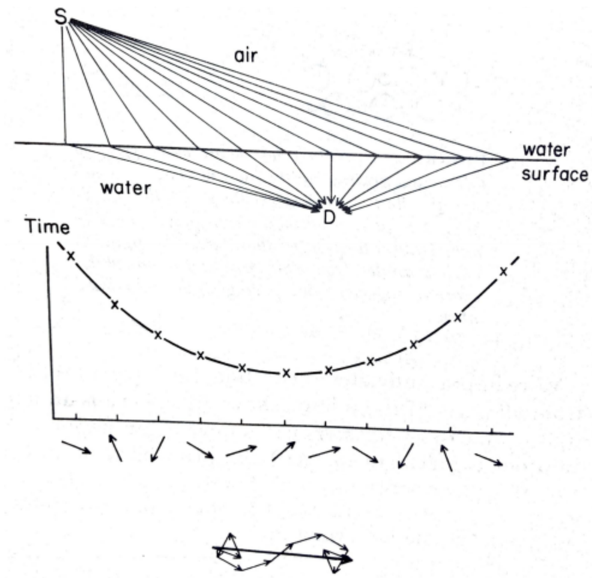
To illustrate that practically no light is reflected by the mirror's edges, consider the left-hand edge of the mirror (corresponding to paths A, B, C). Here, even adjacent paths taken by photons have significant differences in their travel time, so the stopwatches cause the arrows to point in all directions uniformly.



When we take the sum of the paths, we end up where we started, corresponding to no reflection of light.

4.2 Refraction

We can play a similar game to predict the path taken by photons as they cross from air to water:



Again, the largest contribution to the final arrow is due to the paths whose timing does not significantly differ from one another. The accumulation of photons along this path gives us Snell's law, but QED has rendered such laws as "phenomenological" and "macroscopic."

5 Feynman's formulation of quantum mechanics

We will now explore how Feynman formalized the notion of "path" discussed in section (4).[4] This section is meant to illustrate the machinery that drives computational aspects of QED; in section 6, we will return to a conceptual discussion.

Feynman was inspired by one of Dirac's offhand notes:

$$\exp\left(i\int_{t_1}^{t_2}\frac{\mathrm{d}t L_{\text{classical}}(x,\dot{x})}{\hbar}\right) \text{ corresponds to } \langle x_2, t_2 | x_1, t_1\rangle$$

[4] J.J. Sakurai. *Modern Quantum Mechanics*, 2nd ed. Cambridge University Press. 2017. 80-88.

Calling $S(n, n-1) \equiv \int_{t_{n-1}}^{t_n} \mathrm{d}t L_{\text{classical}}(x, \dot{x})$, we can apply successive Dirac-like expressions that somehow contributes to $\langle x_N, t_N | x_1, t_1 \rangle$:

$$\prod_{n=2}^{N} \exp\left(\frac{iS(n, n-1)}{\hbar}\right) = \cdots = \exp\left(\frac{iS(N, 1)}{\hbar}\right)$$

Feynman noted that the classical path is recovered in the $\hbar \to 0$ limit, leading him to conjecture

$$\langle x_N, t_N | x_1, t_1 \rangle \sim \sum_{\text{all paths } (A, B, \dots)} \exp\left(\frac{iS(N, 1)}{\hbar}\right)$$

Inserting a weighting factor $\frac{1}{w(\Delta t)}$ and considering the case that the time interval $t_n - t_{n-1}$ is infinitesimally small,

$$\langle x_n, t_n | x_{n-1}, t_{n-1} \rangle = \frac{1}{w(\Delta t)} \exp\left(\frac{iS(n, n-1)}{\hbar}\right) \tag{3}$$

Focusing our attention on the exponential in equation (3), and assuming the path joining (x_{n-1}, t_{n-1}) to (x_n, t_n) is a straight-line,

$$S(n, n-1) = \int_{t_{n-1}}^{t_n} \mathrm{d}t \left(m\dot{x}^2/2 - V(x)\right) \tag{4}$$

$$= \Delta t \left(\frac{m}{2}\left(\frac{x_n - x_{n-1}}{\Delta t}\right)^2 - V\left(\frac{x_n + x_{n-1}}{2}\right)\right) \tag{5}$$

In the free-particle ($V = 0$) case, the weighting function is[5]

$$\frac{1}{w(\Delta t)} = \sqrt{\frac{m}{2\pi i \hbar \Delta t}}$$

So equation (3) becomes

$$\langle x_n, t_n | x_{n-1}, t_{n-1} \rangle = \sqrt{\frac{m}{2\pi i \hbar \Delta t}} \exp\left(\frac{iS(n, n-1)}{\hbar}\right) \tag{6}$$

To get back $\langle x_N, t_N | x_1, t_1 \rangle$, we define a new integral operator:

$$\int_{x_1}^{x_N} \mathcal{D}(x(t)) = \lim_{N\to\infty} \left(\frac{m}{2\pi i \hbar \Delta t}\right)^{\frac{N-1}{2}} \int \mathrm{d}x_{N-1} \int \mathrm{d}x_{N-2} \dots \int \mathrm{d}x_2$$

giving

$$\langle x_N, t_N | x_1, t_1 \rangle = \int_{x_1}^{x_N} \mathcal{D}(x(t)) \exp\left(i \int_{t_1}^{t_N} \mathrm{d}t \frac{L_{\text{classical}}(x, \dot{x})}{\hbar}\right) \tag{7}$$

[5] using a Gaussian integral with an imaginary argument

Equation (7) is the sum over all possible paths and is known as the Feynman path integral.[6] This is the computational workhorse of quantum field theory and QED.

6 Three Actions of QED

Now that we have seen a conceptual (4) and mathematical (5) description of the quantum "path," we can apply these notions to three basic actions that explain nearly every interaction between light and matter:

6.1 A photon goes from place to place.

If a photon exists at point A, given by the spacetime coordinates (x_1, y_1, z_1) at time t_1, its probability of going to point B, given by the spacetime coordinates (x_2, y_2, z_2) at time t_2, is inversely proportional to the spacetime interval[7]

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - (t_2 - t_1)^2 \tag{8}$$

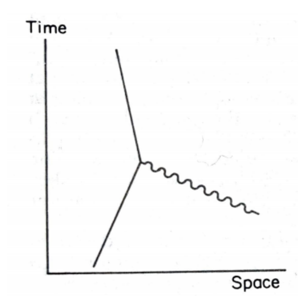
This probability is often denoted $P(A \text{ to } B)$ ("P" for "photon").

6.2 An electron goes from place to place.

If an electron exists at point A, again given by the spacetime coordinates (x_1, y_1, z_1) at time t_1, its probability of going to point B, again given by the spacetime coordinates (x_2, y_2, z_2) at time t_2, is also related to the inverse of equation (8), *as well as* a number that we will call n. This is an empirically found number that helps us match the theory to experiment. This probability is often denoted $E(A \text{ to } B)$ ("E" for "electrons").

6.3 An electron emits or absorbs a photon.

This third action describes the so-called "coupling" between light and matter. The probability of this action happening is described by a single number: j. Feynman used spacetime[8] diagrams to describe this event, denoting electrons with straight lines and photons with wavy lines:



[6]This formulation can be used to derive the Schrödinger equation. For a great presentation of this topic, see Sakurai, 121-122.

[7]Feynman 89.

[8]using one spatial dimension on the horizontal axis, and time on the vertical axis

7 Renormalization

When trying to calculate the values of n and j introduced in section (6), it is necessary to consider all points in space where coupling can occur, "right down to cases where the two coupling points are on top of each other—with zero distance between them."[9] When trying to calculate down to zero distance, however, the equations give meaningless, divergent answers.

To avoid this problem, theoreticians began calculating down to meaningless distances, like 10^{-32} m, which is far below the realm of experimental observation. Schwinger, Tomonaga, and Feynan co-won the 1965 Nobel Prize for independently formalizing this process, calling it "renormalization." Feynman considered this to be a "dippy process" of "hocus-pocus"[10] since it prevents QED from being mathematically self-consistent. A resolution to this mystery is yet to be found.

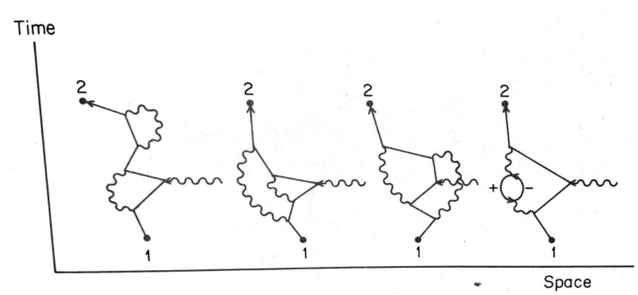
It has been suggested that we cannot use the Standard Model for geometry down to scales like 10^{-32} m. Perhaps gravity, which has not been accounted for in the theory, causes for these infinities. While gravity is the weakest of forces in the physically measureable regime, it becomes important on the 10^{-35} m scale.

8 Measurement of the Magnetic Moment

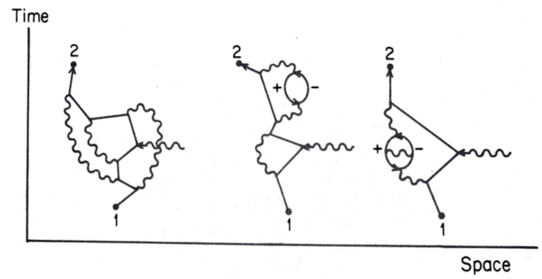
With renormalization in place, calculations were now possible, allowing experimentalists to verify the theory of QED. While there have since been countless verifications of the theory, we will focus on the magnetic moment, which originally plagued Dirac.

The interaction described in section 6.3 is more sophisticated than first meets the eye: there is actually a whole host of ways for an electron to admit or absorb a photon. Since these additional mechanisms (a few of which are outlined below) were not originally accounted for, there was a historic 20+ year-long discrepancy between the theory and experiment.

The probabilities of these events occurring go as powers of j, depending on how many individual internal emission/absorption events occur in the overall process:



[9] Feynman 127.
[10] Feynman 128



Each of the interactions shown above can be thought of as "corrections" to the magnetic moment, making its value slightly greater than 1 (Dirac's original prediction). In all, there are more than 70 possible interactions that explain the emission/absorption of a photon. Accounting for all of these processes (appropriately scaled by factors of j), the theory predicts the magnetic moment to be 1.00115965246, while measurements give 1.00115965221.

Feynman notes: "If you were to measure the distance from Los Angeles to New York to this accuracy, it would be exact to the thickness of a human hair."[11] Since Feynman's day, this error has only decreased, making QED the most accurate physical theory.

References

David Griffiths. *Introduction to Quantum Mechanics*, 2nd ed. Pearson. 2015.

J.J. Sakurai. *Modern Quantum Mechanics*, 2nd ed. Cambridge University Press. 2017.

Richard P. Feynman. *QED: The Strange Theory of Light and Matter.* Princeton Univerity Press. 1985.

[11] Feynman 118.