

Chirag

Sa. “Light”

$$“ \square^2 A^\mu = -\mu_0 J^\mu ”$$

I am amazed my interests align with the meaning of the name my parents gave me. The questions “Who am I?” and “What is light?” are intertwined to me; the latter is as old as humanity itself, and its answer encompasses volumes of human thought.

By passing sunlight through a prism, Isaac Newton showed that the sum of all colors at equal intensity gives white light. This discovery elegantly related the colors of the spectrum, which for centuries were thought to be “fundamentally different” from one another. The story of our understanding of light itself is similar, characterized by the elegant relations between the seemingly disparate.

Electric and magnetic forces were no secret in ancient times: the Greeks were aware of static electricity in the 5th century BCE,¹ and the Chinese aware of Earth’s magnetic field in the 2nd century BCE.² But these forces seemed to have nothing to do with each other: electricity could be observed by rubbing amber on fur; and magnetism was a property intrinsic to Earth. It took Faraday and Maxwell, two millennia later, to respectively show that a changing magnetic field induces an electric field (3), and that by symmetry, a changing electric field induces a magnetic field (4).

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad (1)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (2)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3)$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (4)$$

Previous contributions from Gauss and Ampère showed that in the absence of charges and currents, (1) becomes

$$\vec{\nabla} \cdot \vec{E} = 0 \quad (5)$$

and (4) becomes

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (6)$$

¹“Thales of Miletus,” Encyclopaedia Britannica

²Li Shu-hua (1954). “Origine de la Boussole II. Aimant et Boussolee.”

Taking the curl of (3) and (6),

$$\begin{aligned}\vec{\nabla} \times \vec{\nabla} \times \vec{E} &= -\vec{\nabla} \times \frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \times \vec{\nabla} \times \vec{B} &= \mu_0 \epsilon_0 \left(\vec{\nabla} \times \frac{\partial \vec{E}}{\partial t} \right)\end{aligned}$$

The left-hand-side can be written as ³

$$\begin{aligned}\vec{\nabla} \left(\vec{\nabla} \cdot \vec{E} \right) - \vec{\nabla}^2 \vec{E} &= -\vec{\nabla} \times \frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \left(\vec{\nabla} \cdot \vec{B} \right) - \vec{\nabla}^2 \vec{B} &= \mu_0 \epsilon_0 \left(\vec{\nabla} \times \frac{\partial \vec{E}}{\partial t} \right)\end{aligned}$$

But applying (2) and (5) gives

$$\vec{\nabla}^2 \vec{E} = \vec{\nabla} \times \frac{\partial \vec{B}}{\partial t} \tag{7}$$

$$-\vec{\nabla}^2 \vec{B} = \mu_0 \epsilon_0 \left(\vec{\nabla} \times \frac{\partial \vec{E}}{\partial t} \right) \tag{8}$$

We can simplify further, noting that

$$\begin{aligned}\vec{\nabla} \times \frac{\partial \vec{B}}{\partial t} &= \frac{\partial}{\partial t} \left(\vec{\nabla} \times \vec{B} \right) \\ &= \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}\end{aligned}$$

and

$$\begin{aligned}\vec{\nabla} \times \frac{\partial \vec{E}}{\partial t} &= \frac{\partial}{\partial t} \left(\vec{\nabla} \times \vec{E} \right) \\ &= -\frac{\partial^2 \vec{B}}{\partial t^2}\end{aligned}$$

(7) and (8) then become

$$\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} - \vec{\nabla}^2 \vec{E} = 0 \tag{9}$$

$$\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} - \vec{\nabla}^2 \vec{B} = 0 \tag{10}$$

³Applying the identity $\vec{\nabla} \times \vec{\nabla} \times \vec{P} = \vec{\nabla} \left(\vec{\nabla} \cdot \vec{P} \right) - \vec{\nabla}^2 \vec{P}$

We immediately read off the solutions to (9) and (10) as a spherical wave that travels at speed $\frac{1}{\sqrt{\epsilon_0\mu_0}}$.⁴ This result did more than marry the seemingly unrelated electric and magnetic fields. It expanded Newton's findings (that white light consists of a spectrum) to the entire electromagnetic spectrum, casting everything from radio waves to gamma rays under the same umbrella. It rendered light similar to sound, both of which are characterized by the same parameters (wave numbers, frequencies, phases, amplitudes, etc.). It inspired Einstein to assert that the laws of physics are the same in any inertial frame, a generalization of the Copernican principle that led to a revolutionary understanding of gravity.

How fitting for the study of light, which itself "illuminates," to illuminate our understanding of reality. This is a reality that evidently shares more in common than what we assume. It's as if the various branches of physics are analogous to rivers that all empty into the ocean: the fabric of physics is of the "same stuff," just as rivers and oceans all contain water.

In the spirit of generalizing things, we might as well remove ourselves from physical reality. So many aspects that characterize our lives fit the *rivers : oceans* analogy. Genres of music, media of art, political ideologies, worldviews, religions. . .

The pressure we feel to specialize professionally (or to have a favorite genre, belong to a political party, etc.) is well-founded: we can at least fathom a river. An ecologist can successfully study the local biology of a river without studying the oceans; we similarly do physics (and live) "successfully" without often thinking of the "big picture." But my name begs me to take some more time to do otherwise.

I may never have the means to describe what I make of this "big picture," but the reward is in the journey. The beauties of life—physics, music, friendship—will continue to draw me deeper.

⁴It is of course coincidental that my name starts with the letter $c \equiv \frac{1}{\sqrt{\epsilon_0\mu_0}}$, but it still means something to me.