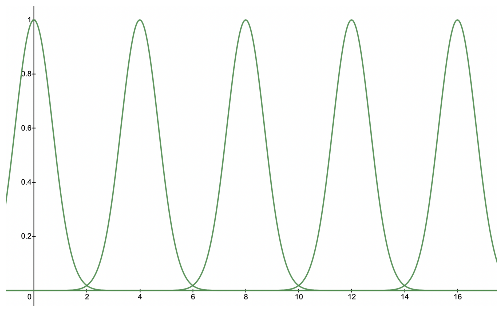
Practice problem

Consider an underwater[1] spherical wave that oscillates in time as shown below:



This "Gaussian comb" profile is defined over one period 4τ, where $\tau = 0.01$ s, as

$$p = p_0 e^{-(t-4\tau)^2}$$

The pressure amplitude p_0 of the wave is 50 Pa when measured 1 m from the source.

1. Calculate the sound pressure level 200 m from the source. You may assume that

$$\int_{-2\tau}^{2\tau} e^{-Cx^2}\,\mathrm{d}x \simeq \int_{-\infty}^{\infty} e^{-Cx^2}\,\mathrm{d}x = \sqrt{\frac{\pi}{C}}$$

The sound pressure level at $r = 200$ m is

$$\mathrm{SPL}(r = 200) = 20\log\left(\frac{p_{\mathrm{rms}}(r = 200)}{10^{-6}}\right)$$

To find $p_{\mathrm{rms}}(r = 200)$, we must first find p_0, the amplitude of the pressure, at 200 m:

$$\frac{p_0(r = 200)}{p_0(r = 1)} = \frac{1}{200}$$
$$\frac{p_0(r = 200)}{50} = \frac{1}{200}$$
$$\implies p_0(r = 200) = 0.25 \text{ Pa}$$

So the waveform 200 m away from the source is $p = 0.25e^{-(t-4\tau)^2}$, from which we can calculate p_{rms}:

[1] $\rho_0 = 1026$ kg/m^3, 1500 m/s, $\rho_0 c_0 = 1.54$ Mrayls

$$
\begin{aligned}
p_{\text{rms}} &= \left(\frac{1}{t_{\text{av}}}\int_{t_{\text{av}}} |p|^2 \,\mathrm{d}t\right)^{\frac{1}{2}} \\
&= \left(\frac{1}{4\tau}\int_{-2\tau}^{2\tau} \left|0.25e^{-(t-4\tau)^2}\right|^2 \mathrm{d}t\right)^{\frac{1}{2}} \\
&\simeq \left(\frac{1}{4\tau}\int_{-\infty}^{\infty} \left|0.25e^{-(t-4\tau)^2}\right|^2 \mathrm{d}t\right)^{\frac{1}{2}} \\
&\simeq 0.25\left(\frac{1}{4\tau}\int_{-\infty}^{\infty} e^{-2(t-4\tau)^2} \,\mathrm{d}t\right)^{\frac{1}{2}} \\
&\simeq \frac{0.135}{\sqrt{\tau}}\left(\sqrt{\frac{\pi}{2}}\right)^{\frac{1}{2}} \\
&\simeq \frac{0.135}{\sqrt{0.01}}\left(\frac{\pi}{2}\right)^{\frac{1}{4}} \simeq 1.51
\end{aligned}
$$

So the sound pressure level is

$$
\text{SPL} = 20\log\left(\frac{1.51}{10^{-6}}\right) = 123.58 \text{ dB}
$$

2. Calculate the particle velocity amplitude u_0 200 m from the source.

   Since $|Z| = \frac{p_0}{u_0}$,

$$
u_0 = \frac{p_0}{|Z|}
$$

   Recall that the impedance is $Z = \frac{\rho_0 c_0}{1+1/jkr}$, and the magnitude of the impedance is $\frac{\rho_0 c_0 kr}{\sqrt{1+k^2r^2}}$.

$$
\begin{aligned}
u_0 &= \frac{p_0}{\frac{\rho_0 c_0 kr}{\sqrt{1+k^2r^2}}} \\
&= \frac{0.25}{\frac{1026*1500*(2\pi*0.04/1500)*200}{\sqrt{1+(2\pi*0.04/1500)^2*200^2}}} \\
&= 4.85\ \mu\text{m/s}
\end{aligned}
$$

3. Suppose the source is Dr. Hamilton's pet hermit crab, Hammy, maniacally laughing at the world as it sits on the ocean floor at the edge of a continental shelf.[2] Calculate the sound power level.

[2] That is, the sound propagates in only one quadrant of a sphere

The sound power level is given by

$$\mathrm{PWL} = 10\log(W/10^{-12})$$

where

$$W = \frac{1}{4}(4\pi r^2 I) = \pi r^2 I$$

since the sound is radiated in only one quadrant of a sphere. I found using the exact result of the intensity of a spherical wave:

$$I = \frac{p_{\mathrm{rms}}^2}{\rho_0 c_0} = \frac{1.51^2}{1026 * 1500} = 1.48 * 10^{-6}$$

So $W = 0.186$ W, and

$$\mathrm{PWL} = 10\log(0.186/10^{-12}) = 112 \text{ dB}$$