Potential Topics for Video Project

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1 Napoleon's Theorem

Statement of the theorem[1]

If we construct outward-pointing equilateral triangles on the three sides of an arbitrary given triangle, then the triangle formed by the centroids of the three equilateral triangles is equilateral.

Motivation

This theorem fits the criteria of the assignment, as it is challenging to prove but can be reduced to first principles. It is somewhat reminiscent of Morley's theorem, which says that the trisectors of any triangle meet to form the vertices of an equilateral triangle.

Approach

To prove this theorem, we would appeal to vectors and linear operators. While we are unsure as to whether such topics would be covered in the course, they are of personal interest and would be rewarding to study.

2 Pappus's Theorem

Statement of the theorem[2]

Suppose that points A, B, and C lie on some line l and that points X, Y, and Z lie on line m, where the six points are distinct and the two lines are also distinct. Assume that lines BZ and CY meet at P, lines AZ and CX meet at Q, and lines AY and BX meet at R. Then points P, Q, and R are collinear.

[1] Isaacs, Theorem 5.14
[2] Isaacs, Theorem 4.16

Motivation

This theorem is hard to believe, as it is generally true for six generic, distinct points on two randomly oriented lines l and m. It also invokes several other important theorems, including the theorem of Menelaus. We would therefore be proving Menelaus's theorem in our introduction.

Approach

We would employ an approach laid out by Isaacs in section 4D of the text. This approach draws heavily on previous topics covered in chapter 4, as mentioned, so an thorough introduction would be given.

3 Van Schooten's theorem

Statement of the theorem[3]

For an equilateral triangle ΔABC with a point P on its circumcircle, the length of longest of the three line segments PA, PB, PC connecting P with the vertices of the triangle equals the sum of the lengths of the other two.

Motivation

The claim of this theorem is very elegant and is reminiscent of the theorem of Pythagoras. The theorem itself is a result of Ptolemy's theorem, so we would also prove Ptolemy's theorem in our introduction.

Approach

We would follow an approach outlined on the corresponding Wikipedia page. It is not hard to get from the statement of Ptolemy's theorem to the statement of Van Schooten's theorem, so a majority of the presentation would be devoted to proving Ptolemy's theorem.

4 Butterfly Theorem

Statement of the theorem

Suppose that chords PQ and RS of a given circle meet at the midpoint M of chord Ab. If X and Y are the points where PS and QR meet AB, respectively, then $XM = YM$ [4]

[3] from Wikipedia

[4] Isaacs, theorem 3.13

Motivation

This is a comprehensive proof that demands an understanding of many of the topics covered in the course, including similarity, vertical angles, inscribed figures, and measures of arcs. It would be a nice way to review and simultaneously apply the topics covered in the course.

Approach

We will follow Isaac's proof.

5 Pitot's Theorem

Statement of the theorem[5]

In a tangential quadrilateral (i.e. one in which a circle can be inscribed) the two sums of lengths of opposite sides are the same. Both sums of lengths equal the semiperimeter of the quadrilateral.

Motivation

This theorem involves several of the topics discussed in our class, including tangent lines, inscribed quadrilaterals, and semiperimeters.

Approach

We will follow the general argument described *here*. Our goal would be to provide the proof of both the theorem and its converse.

[5] from Wikipedia

An additional option...

6 Casey's theorem

Statement of the theorem[6]

Let O be a circle of radius R. Let O_1, O_2, O_3, O_4 be (in that order) four non-intersecting circles that lie inside O and tangent to it. Denote by t_{ij} the length of the exterior common of the circles O_i, O_j Then

$$t_{12} \cdot t_{34} + t_{14} \cdot t_{23} = t_{13} \cdot t_{24}$$

Motivation

This theorem is a more sophisticated statement involving many of the geometric figures studied in this course, including circles, inscribed quadrilaterals, bitangent/tangent lines, etc. In invokes the Pythagorean theorem (for which we will present a fun proof) and represents a generalization of Ptolemy's theorem (which is really the degenerate case in which the circles O_1, O_2, O_3, O_4 are reduced to points P_1, P_2, P_3, P_4

Approach

We will follow the approach outlined on the corresponding Wikipedia page.

[6] from Wikipedia